

Name _____ Date _____

Partner _____

Euler's Method: Mechanics Lab #2

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Objective: to learn a technique for predicting an object's motion and to become comfortable with Excel spreadsheets.

Equipment: Computer with Excel, personal critical thinking skills

Background: Consider an object which is dropped in a gravitational field. It falls with constant acceleration (if air resistance is small).

On Earth the acceleration of gravity is _____.

On the Moon the acceleration is about 1/6 of that value, so is _____.

The object's velocity builds up every second from the acceleration, and the position changes also from the velocity each second. We can make a mathematical model of this progression using small discrete time steps. For each time step of length dt , the equations which express the kinematics are

$$\text{acceleration} = a = \text{constant} = \text{_____} \text{ (on Earth)}$$

$$\text{velocity} = v = v_i + a dt$$

$$\text{position} = s = s_i + v dt$$

The starting conditions are v_0 as the velocity and s_0 as the position of the object, at time = 0. Also, v_i is the initial velocity for a time step and s_i is the initial position for a time step.

If we let the time step be a real number Δt rather than a tiny infinitesimal quantity dt , then we have changed a differential equation into a finite difference equation. This is Euler's method. It uses the rate of change at the beginning of each time step to move the calculated values forward in time. Today we will use $\Delta t = \text{time step} = 0.01$ seconds.

In Excel we can refer to a constant parameter such as the acceleration of gravity by using dollar signs in the parameter's cell name. For instance, use $\$B\7 instead of just B7 to indicate a constant. Multiplication is shown as an asterisk *.

Predictions: Make sketch graphs of the acceleration vs. time, the velocity vs. time, and the position vs. time for an object which falls with these starting conditions (scenarios I, II, III, IV):

I. Gravity = Earth, $v_0 = 0$, $s_0 = 0$

II. Gravity = Earth, $v_0 = 2$, $s_0 = 0$

III. Gravity = Moon, $v_0 = 2$, $s_0 = 0$

IV. Gravity = Earth, $v_0 = 2$, $s_0 = 0$, air resistance is fierce

Summary:

- a). The Problem: For each of these scenarios write a sentence describing the problem.
- b). The Motion: Consider scenario I as the standard scenario. For each of the scenarios II, III, and IV write a sentence expressing how the motion of the object differs from the motion in scenario I.

Procedure:

1. Work in teams of two students. Open a spreadsheet. Please follow the directions carefully. Headings: In column A in the first several lines type your names, Mechanics Lab #2: Euler's Method: Freefall, Today's date.

Save the spreadsheet to the proper physics folder in My Documents.

Skip a line.

2. Titles: In column A type "Scenario:"

In column A in the next four rows downwards type these headings: g:, v0:, s0:, delta_time:, air_resistance:.

In column A, B, and C type Time step, Velocity, Position.

Below these put n, vn, sn.

3. Time equation: Under n put 0, then under that put $=(\text{cell number above}) + 1$. Fill down until n is 50.

4. First scenario: In column B in the correct rows put "I, on Earth", -9.8 , 0 , 0 , 0.01

Under vn put $=(\text{cell number where you stored the initial velocity number})$.

Under that put $=(\text{cell number above}) + (\text{cell number where you stored the gravity number, with } \$\$) * (\text{cell number where you stored the delta_time number, with } \$\$)$.

Fill down until the n = 50 row is reached. (You can fill down quickly by double clicking on the lower right-hand corner of the cell with the formula in it.)

Under sn put $=(\text{cell number where you stored the initial position number})$.

Under that put $=(\text{cell number above}) + (\text{cell number of the velocity for this step}) * (\text{cell number where you stored the delta_time number, with } \$\$)$.

Fill down until the n=50 row is reached.

Save again. (Save every minute)

5. Graphs: Now make a graph of velocity vs. time by selecting these numbers, selecting Insert/Scatter Graph/with unconnected points.

Choose gridlines (major x gridlines as well as major y gridlines), delete the legend "Series 1", make appropriate titles for the chart and for the x and y axes.

Move this graph to the right of all the number columns, please, and make it smaller.

Does this graph match what you predicted above? _____

Make a graph of position vs. time.

Does this graph match what you predicted above? _____

6. Further scenarios: For each scenario make two new columns, one for velocity and another one for position by copying (CTRL c) and pasting (CTRL v) the titles and first data values from your columns B and C into column D, F, and H. This time make the input values above the velocity column to be "II, on Earth", -9.8 , 2 , 0 , 0.01 , then "III, on the Moon, etc.

Correct the first equations carefully, then fill down and then make the graphs. Note that when the data columns you want to graph are separated, you must hold down the CTRL key while selecting the second column.

Do these graphs match what you predicted for scenario II above? _____

Do these graphs match what you predicted for scenario III above? _____

7. Including Air Resistance: To include air resistance for scenario IV we need to have the acceleration include a term related to the velocity itself. Put $v_n = (\text{previous } v) + (\text{gravity} - \text{air_resistance_constant} * \text{previous } v) * \text{delta_time}$. Try different values of the air_resistance_constant until you see strong terminal velocity behavior.

Hint: put the value for this constant in a cell and refer to it with $\$$. Then you can change the constant easily and watch the graphs change right away.

8. Make the spreadsheet pretty by putting all the small graphs on the right hand page (page 3). Look at Print Preview and adjust spacings so that the numerical columns fit on only two pages. Print a copy of the spreadsheet with small graphs for each partner. E-mail a copy of the spreadsheet to each partner.

Conclusions:

Future work in this field:

SAMPLE

Mary Lou West

Mechanics Lab #2: Euler's Method: Freefall

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Scenario:	I, on Earth	II, on Earth	III, on Moon	IV, on Earth, air resistance		
g:	-9.8	-9.8	-1.63	-9.8		
v0:	0	2	2	2		
s0:	0	0	0	0		
delta_time:	0.01	0.01	0.01	0.01		
air resistance:				5		
Time step	Velocity Position		Velocity Position		Velocity Position	
n	vn	sn	vn	sn	vn	sn
0	0	0	2	0	2	0