

Imaging with $\tau = 0$ versus $t = 0$: implications for the inverse scattering internal multiple attenuation algorithm

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Summary

The inverse scattering subseries for removing free surface and internal multiples provided the first comprehensive theory for removing multiples from an arbitrary heterogeneous earth without any subsurface information whatsoever. Furthermore, taken as a whole, the inverse series provides a fully inclusive theory for processing both primaries and multiples directly in terms of an inadequate velocity model, without updating or in any other way determining the accurate velocity configuration. Hence, the inverse series and, more specifically, its subseries that perform imaging and inversion of primaries, has the potential to allow processing primaries to catch up to processing multiples in concept and effectiveness.

As the capability for processing primaries is advancing and the models for testing/evaluation become more complex, our fundamental definition/understanding of what we call primaries and multiples needs to expand to include a wider set of event types as well. Expanding the type of arrivals we consider primaries is important for imaging and inversion and also, as subevents, for internal multiple attenuation algorithms. This research pushes forward the description/processing of headwave arrivals as prime events (primaries) or sub-events of composite events (multiples) and it is part of our effort to accommodate broader data types and events in the recorded data.

Introduction

The inverse scattering series is a multi-dimensional inversion procedure that directly determines physical properties using only recorded data and a reference medium. The inversion process can be thought of as performing the following four tasks: (1) free surface multiple removal, (2) internal multiple removal, (3) location of reflectors in space and (4) identification of medium property changes across reflectors. These tasks were associated with subseries of the full series, subseries which, if identified, would perform their job as if no other task existed in the series. Two immediate advantages of this separation of tasks are the favorable convergence properties of the subseries and the ability to judge the effectiveness of each step before proceeding on to the next. Since the entire process requires only data and reference medium information, it is reasonable to assume that intermediate steps that are associated with achieving that objective would also be attainable with only the reference medium and data.

Subseries that exhibit this property have been identified for all four tasks (Weglein et al. 2003 and references therein). Algorithms resulting from the subseries

for the task of free surface and internal multiple attenuation have been successfully applied to field data (Weglein et al. 2003). Algorithms resulting from imaging and inversion subseries are still in the testing/evaluating stages with the realism of the model increasing with each test (Shaw and Weglein 2004, Zhang and Weglein 2004, Fang et al. 2004). As the capability for processing primaries is advancing and the models become more complex, our fundamental definition/understanding of what we call primaries needs to expand as well. The inverse series can accommodate a wide class of model-types and the point scatterer formalism is unrestrictive in the sense that, in principle, it can understand and expect data containing reflection, diffraction and refraction events. This paper is a first step towards examining the inclusion of headwaves into the imaging and internal multiple attenuation algorithms derived from the inverse scattering series. For this purpose, we discuss the $t = 0$ versus the $\tau = 0$ imaging condition and show that the latter is more general in its ability to image events that exhibit a horizontal propagation part in their path, e.g. headwaves along a horizontal interface.

The idea of using $\tau = 0$ as an imaging condition has been used by Clayton and McMechan (1981) to image refraction data to produce velocity-depth profiles from recorded data. Their method involves a slant-stack of the data to produce a wavefield in the $p - \tau$ domain, where p is the horizontal slowness, and a downward continuation and imaging with $\tau = 0$.

A comparison between travel time t and intercept time τ

The imaging concept in the $f - k$ migration procedure, assumes that the turning point of the wavefield is a point in space and hence, by restricting the time t to zero, one would obtain the location where an up-going wave would co-exist with the first arrival of a down-going wave, hence

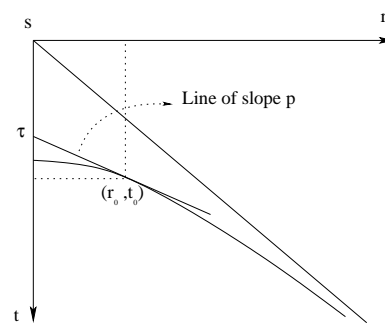


Fig. 1: The definition of the vertical time τ .

Imaging with $\tau = 0$ versus $t = 0$

the position in space of the reflector. While this is true for regular reflections, it is not true for events which contain horizontal parts in their propagation paths. The turning point of a headwave is not a point in space but an entire linear horizontal propagation. In consequence, an imaging condition $t = 0$ on data which contains these kind of events would interpret them as regular reflections and hence it would "create" reflectors at wrong depths to accommodate them. In this section we describe the connection and differences between the travel time t and the intercept time τ and show that the imaging condition $\tau = 0$ is a generalization of $t = 0$ which would image the headwaves from horizontal interfaces at the correct depth.

By definition, the intercept or vertical time τ of an event arriving at offset r_0 in travel-time t_0 is the vertical component of the travel-time t_0 or, in other words, t_0 projected to zero offset along a line of slope p through the point (r_0, t_0) (see Figure 1). The slope p of the tangent to the curve representing a reflection in the shot record pictured can be calculated as

$$p = \left(\frac{dt}{dr} \right)_{(r_0, t_0)} \quad (1)$$

and hence it represents the horizontal slowness associated with that particular arrival. The equation of the tangent line gives a relationship between the travel-time t and the vertical time τ of a particular arrival

$$t = \tau + pr. \quad (2)$$

This formula represents a decomposition of the total time into a horizontal time, pr , and a vertical time τ . To better understand this decomposition, consider a plane-wave component of constant horizontal slowness p of a wavefield produced by a 3D point source and moving through a medium with constant velocity c_0 . The points of intersections of this planewave with the horizontal interface and the vertical line, move along them with constant respective speeds c_H and c_V such that

$$\sin i = \frac{c_0}{c_H} \quad (3)$$

and

$$\cos i = \frac{c_0}{c_V} \quad (4)$$

(see Figure 2a). The horizontal and the vertical slowness, p and q , are defined as

$$p = \frac{1}{c_H} = \frac{\sin i}{c_0} \quad (5)$$

and

$$q = \frac{1}{c_V} = \frac{\cos i}{c_0} \quad (6)$$

and they are related through

$$p^2 + q^2 = \frac{1}{c_0^2}. \quad (7)$$

Notice (from Figure 2b) that for the plane-wave to go from the source s to the incidence point I in time t is equivalent for the projection point onto the vertical to go from s to A along the vertical with the speed c_V and for the projection point onto the horizontal to go from A to I along the interface with the speed c_H . In this way the total time t is decomposed into two parts, a vertical and a horizontal time corresponding to the horizontal and vertical motion of the projection points as follows

$$t = \frac{z_1}{c_V} + \frac{r}{c_H} = qz_1 + pr = \tau + pr. \quad (8)$$

From the previous equation (8), the condition $t = 0$ always implies $\tau = 0$. The converse is not true. For a regular reflection $\tau = 0$ does imply that $t = 0$. To show this notice that $\tau = 0$ implies that no vertical propagation takes place, hence $z_1 = 0$. However for this type of event there is a relationship between the horizontal and the vertical coordinates, r and z , namely

$$z \tan i = r \quad (9)$$

and so $z_1 = 0$ implies $r = 0$ and together imply $t = 0$. The same statement (and argument to prove it) applies to other events for which there is a similar relationship between the horizontal and the vertical coordinates (for example for a turning wave). However this is not true for all seismic events. For example, for events that contain horizontal parts in their propagation paths, e.g. a headwave from a horizontal interface, there is no relationship between r and z ; in fact, for the part where the ray travels horizontally, we have $z = 0$ (no vertical propagation) while $r \neq 0$. In this case it is obvious that $\tau = 0$ does not imply $t = 0$.

The result we want to emphasize is that

$$t = 0 \Rightarrow \tau = 0 \quad (10)$$

$$\tau = 0 \not\Rightarrow t = 0 \quad (11)$$

which implies that the imaging condition $\tau = 0$ is more general than $t = 0$. To image with $\tau = 0$ means to consider only the up-down motion of the wave and disregard any horizontal displacement that it might have (see Figure 3). This way, headwaves from horizontal interfaces are regarded as a down-up motion, rather than the down-lateral-up, and the imaging condition seeks the

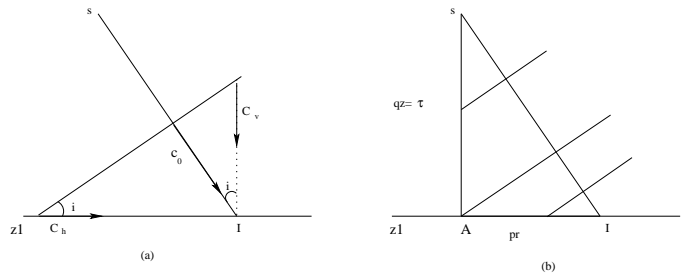


Fig. 2: The relationship between t and τ .

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point where the wave turns, i.e. it changes from a downward into an upward propagation.

Analytic example of imaging headwaves with $\tau = 0$

Consider a 3D acoustic experiment with source and receiver located at the same depth $z = 0$ and one horizontal interface located at depth z_1 separating two media with wave propagation velocities c_0 and c_1 . The media are assumed to have no lateral variation. The post-critical data in such an experiment is (see e.g. Aki and Richards Ch. 6)

$$P = P^R + P^H \quad (12)$$

where, in the above equation, all quantities are functions of receiver coordinates x_g, y_g , source coordinates x_s, y_s and temporal frequency ω , and

$$P^R = \frac{R}{d} \exp(ik_r r + i\nu_0 2z_1) \quad (13)$$

is the reflected event and

$$P^H = A(\omega) \exp\left(i\omega \frac{r}{c_1} + i\nu_0 2z_1\right) \quad (14)$$

with $A(\omega) = \frac{i}{\omega} \frac{c_0^2}{(1-c_0^2/c_1^2)} \frac{1}{r^{1/2} L^{3/2}}$, is the headwave. In these expressions R is the angle-dependent reflection coefficient, d is the total distance from the source to reflection point to receiver, r is the horizontal offset and satisfies $r = \sqrt{(x_g - x_s)^2 + (y_g - y_s)^2}$, k_r is its conjugate in the K-space domain, ν_0 is the vertical wavenumber of the first medium and satisfies $\nu_0^2 + k_r^2 = \omega^2/c_0^2$, and L is the length of the horizontal part of the ray representation of the headwave. Downward continue both the source and receiver to same arbitrary depth and obtain

$$P(z) = P^R(z) + P^H(z) \quad (15)$$

where

$$P^R(z) = \frac{R}{d} \exp(ik_r r + i\nu_0 2(z_1 - z)) \quad (16)$$

and

$$P^H(z) = A(\omega) \exp\left(i\omega \frac{r}{c_1} + i\nu_0 2(z_1 - z)\right). \quad (17)$$

and the dependence on the other variables has been omitted from writing. The reflection can be easily imaged with the $t = 0$ imaging condition to obtain the reflectivity of the reflection point at the correct depth z_1 . The same procedure, applied to the part of the data representing the headwave, would assume that the turning point of that event is a point in space and it would seek that point hence creating an image at the wrong depth. To image the headwave with $\tau = 0$ we first inverse Fourier Transform to bring the data back to the time domain. However, ω is conjugated to the travel time t and we want to bring

the data back to the vertical time τ domain where we can apply the imaging condition. We define the image I to be

$$I(\tau) = \int d\Omega e^{-i\Omega\tau} P^H(z) \quad (18)$$

where

$$\Omega = \omega \left(1 + \frac{r}{c_1\tau}\right). \quad (19)$$

With the full expression for P^H we have

$$I(\tau) = \int d\Omega e^{-i\Omega\tau} A(\omega) \exp\left(i\omega \frac{r}{c_1} + i\nu_0 2(z_1 - z)\right). \quad (20)$$

By substituting the expression for Ω we obtain

$$I(\tau) = \int d\Omega e^{-i\omega\tau} A(\omega) \exp(i\nu_0 2(z_1 - z)) \quad (21)$$

and after imaging with $\tau = 0$ we find

$$I(\tau = 0) = \int d\Omega A(\omega) \exp(i\nu_0 2(z_1 - z)). \quad (22)$$

This last expression represents a delta like event at the correct depth z_1 hence showing that the headwave is imaged correctly. Notice that the new condition discards any horizontal propagation (and time associated with it) and only takes into consideration down-up propagations, as one can also see from Figure 3. For the headwave this means discarding the horizontal propagation along the interface. It is not difficult to see that the procedure outlined above also images the reflection data at the correct depth.

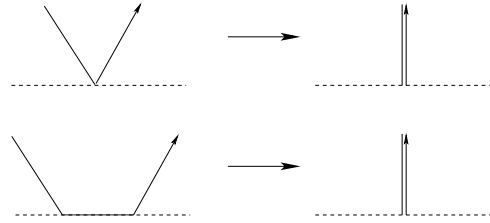


Fig. 3: Events in t and τ : the left column shows a reflection and a headwave in traveltime t ; the right column shows the same event in the vertical time τ

Internal multiple attenuation algorithm derived from the inverse scattering series

The second term in the inverse scattering subseries for internal multiple attenuation is (see e.g. Weglein et al 2003)

$$b_3 = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 e^{-iq_1(\epsilon_g - \epsilon_s)} dk_2 e^{iq_2(\epsilon_g - \epsilon_s)} \\ \times \int_{-\infty}^{\infty} dz_1 e^{i(q_g + q_1)z_1} b_1(k_g, k_1, z_1)$$

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$$\begin{aligned} & \times \int_{-\infty}^{z_1} dz_2 e^{i(-q_1 - q_2)z_2} b_1(k_1, k_2, z_2) \\ & \times \int_{z_2}^{\infty} dz_3 e^{i(q_2 + q_s)z_3} b_1(k_2, k_s, z_3) \end{aligned} \quad (23)$$

where $z_1 > z_2$ and $z_2 > z_3$ and b_1 is defined in terms of the original pre-stack data with free surface multiples eliminated, D' , to be

$$D'(k_g, k_s, \omega) = (-2iq_s)^{-1} B(\omega) b_1(k_g, k_s, q_g + q_s) \quad (24)$$

with $B(\omega)$ being the source signature. The terms $b_1(k_g, k_s, z)$ in formula (23) can be thought of as being obtained through the following procedure. Start with the effective data in the $f - k$ domain, $b_1(k_g, k_s, \omega)$, and downward continue the source and receiver by applying a phase-shift $e^{ik_z z} b_1(k_g, k_s, \omega)$. Subsequent integration over k_z to obtain $b_1(k_g, k_s, z)$ is a simple Jacobian away from integration over ω ($t = 0$ imaging condition). Hence, the algorithm can be interpreted as a sequence of un-collapsed migrations restricted to lower-higher-lower pseudo-depths. After imaging with $t = 0$ every event in the recorded data set is considered a sub-event, i.e., a component of a composite internal multiple.

The comparison between the $t = 0$ and $\tau = 0$ imaging conditions shows that the latter is capable of imaging a larger class of events, e.g. headwaves from horizontal interfaces. Imaging this type of arrival is important not only from the point of view of prime events as information carriers, but also (as we saw above) from the point of view of sub-events of composite events, e.g. in removing internal multiples with a refracted subevent. This research is a first step towards examining the inclusion of headwaves into the imaging and internal multiple attenuation algorithms derived from the inverse scattering series.

Conclusions

The purpose of this paper is to present the advantages of imaging with $\tau = 0$ versus imaging with $t = 0$. The former condition is a generalization of the latter which also has the ability to image headwaves from horizontal interfaces at the correct depth. This research is part of our effort to better understand and accommodate broader data types and events in the recorded data for the imaging and inversion of primaries and the removal of internal multiples.

Acknowledgments

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